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## ABSTRACT

We report the interaction between active non-spherical swimmers and a long-standing flow structure, Lagrangian coherent structures (LCSs), in a weakly turbulent two-dimensional flow. Using a hybrid experimental–numerical model, we show that rod-like swimmers have a much stronger and more robust preferential alignment with attracting LCSs than with repelling LCSs. Tracing the swimmers' Lagrangian trajectories, we reveal that the preferential alignment is the consequence of the competition between the intrinsic mobility of the swimmers and the reorientation ability of the strain rate near the attracting LCSs. The strong preferential alignment with attracting LCSs further leads to a strong accumulation near the attracting LCSs. Moreover, we show the self-similarity of this accumulation, which reduces the intricate interaction to only one control parameter. Our results generically elucidate the interaction between active and non-spherical swimmers with LCSs and, thus, can be widely applied to many natural and engineered fluids.

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## I. INTRODUCTION

Organisms ranging from bacteria to protozoa, crustacean, and fish are ubiquitous in both natural<sup>1,2</sup> and engineered fluids.<sup>3,4</sup> These organisms are universally featured with non-spherical shapes and considerable intrinsic mobility, and they are usually surrounded by complex or even turbulent fluid environments. Both the aspect ratio and swimming speed of aquatic organisms have wide ranges.<sup>5,6</sup> Table I lists the typical aspect ratio and swimming speed of some common species. The interactions between these active non-spherical swimmers and dynamic fluid environments are essential in a wide range of environmental processes, such as nutrient uptake,<sup>7,8</sup> infection,<sup>9</sup> marine predator foraging,<sup>10</sup> and marine megafauna bycatching.<sup>11</sup> It is also crucial in medical and biological applications such as designing microrobots and artificial swimmers.<sup>12,13</sup> Unlike passive spherical tracers, active non-spherical swimmers do not exactly follow the flow due to their shapes and intrinsic mobility.<sup>7,14,15</sup>

Swimmers' mobility and shape play important roles in the transport of swimmers in fluid environments. The mobility of the swimmers causes them to interact with dynamical structures in the flow in a different way from passive tracers, and such different microscopic interactions can result in different macroscopic behaviors.<sup>23</sup> It

is found that the mobility of swimmers does not necessarily lead to enhanced transport of them in a fluid environment, and small mobility can result in reduced transport.<sup>24</sup> The shape of the swimmers can lead to preferential alignments with flow velocity<sup>25</sup> and repelling Lagrangian coherent structures (LCSs) of hyperbolic fixed points.<sup>23</sup> Researchers also reveal that the intrinsic mobility and the shape factor of swimmers conspire to enhance transport.<sup>26</sup> In addition, it is found that the hydrodynamic shear<sup>27</sup> and flow velocity<sup>25</sup> can have nontrivial impacts on the orientation and spatial heterogeneity of swimmers resulting from their shapes and intrinsic mobility, in which the fluid shear can trap swimmers<sup>27</sup> and rod-like swimmers preferentially align with flow velocity.<sup>25</sup>

Turbulence can also lead to the heterogeneous distribution of swimmers and particles.<sup>28–34</sup> In isotropic turbulence, non-spherical swimmers preferentially sample regions of lower fluid vorticity and accumulate in those regions.<sup>30</sup> The clustering and patchiness are much weaker in isotropic turbulence than that observed in simple cellular or vortical flows<sup>24,26</sup> due to the complex topology.<sup>35</sup> For gyrotactic swimmers that have a preferred swimming direction, it is found that the coupling of swimmer mobility and shear can cause clustering in horizontal thin layers<sup>36</sup> and microscale patchiness,<sup>28</sup> and spherical swimmers show a stronger clustering than elongated swimmers in

**TABLE I.** Typical aspect ratios and swimming speeds of some common aquatic organisms. The aspect ratio of *Menidia menidia* is estimated through the data obtained from [www.fishbase.us](http://www.fishbase.us).

| Species/genus                              | Common name         | Aspect ratio ( $\gamma$ ) | Swimming speed (cm/s) |
|--------------------------------------------|---------------------|---------------------------|-----------------------|
| <i>Escherichia coli</i> <sup>16,17</sup>   | E. coli             | 2 – 6                     | 0.0014                |
| <i>Paramecium</i> <sup>18,19</sup>         |                     | $\sim 2$                  | $\sim 0.2$            |
| <i>Euphausia pacifica</i> <sup>20,21</sup> | North Pacific krill | $\sim 7$                  | 2.5                   |
| <i>Menidia menidia</i> <sup>22</sup>       | Atlantic silverside | $\sim 6$                  | 2.3                   |

isotropic turbulence.<sup>35</sup> It is also found that elongation as a consequence of chain formation can enhance the vertical migration of motile phytoplankton through weak to moderate turbulence.<sup>37</sup>

Despite the substantial contribution in the past, several unanswered questions deserve further investigation. First, a comparison between the effects of repelling LCSs and attracting LCSs on active non-spherical swimmers is not known. Second, while the increasing accumulation near the stable manifold has been presented,<sup>23</sup> the mechanism of such accumulation remains a mystery. Third, how does the accumulation strength near the LCSs changes as a function of a wide range of swimming speeds and aspect ratios is not clear. Finally, many of the previous studies were based on simulations with time independent flows (such as cellular flow); an investigation of LCSs' effect on swimmers in time dependent and turbulent flow is needed.

Here, we tackle the above-mentioned questions using a hybrid experimental–numerical model. First, we find that the alignment of non-spherical swimmers with attracting LCSs is one order of magnitude stronger than that with repelling LCSs. Second, we reveal the Lagrangian history of swimmers before they align with the attracting LCSs and claim that the preferential accumulation is due to the competition between the intrinsic mobility of the swimmers and the reorientation ability of the strain rate near the attracting LCSs. Moreover, we show the accumulation concentration map in the parameter space spanned by the intrinsic parameters of the swimmer. Finally, we find the self-similarity of the accumulation near the attracting LCSs in the parameter space, which indicates that the interaction between swimmers and attracting LCSs can be reduced to only a single control parameter.

We begin below by first describing the method of measuring the flow field and the hybrid experimental–numerical model in Sec. II. In Sec. III, we show the preferential alignment of active non-spherical swimmers with attracting LCSs. The heterogeneous distribution of active non-spherical swimmers and the self-similarity are discussed in Sec. IV. Finally, in Sec. V, we summarize our results and draw conclusions.

## II. EXPERIMENTS AND METHODS

### A. Measurement of the flow field

The flow field used in this work was generated with an electro-magnetically driven thin-layer flow system and measured using particle tracking velocimetry. Details for both apparatus and particle tracking velocimetry have been described in other works using similar methods.<sup>38–40</sup> The schematic of the experimental setup is shown in Fig. 1. Experiments were performed with a 0.5 cm layer solution of 14% by mass NaCl in water, with density  $\rho = 1.101 \text{ g/cm}^3$ . This fluid lies on a glass floor coated with hydrophobic wax in order to reduce

friction. Beneath the glass floor, a square array of cylindrical neodymium–iron–boron permanent magnets spaced  $L_m = 2.54 \text{ cm}$  on the center are arranged in stripes of alternating polarity. The magnets have diameters of 1.27 cm and thicknesses of 0.32 cm. The magnet strength is roughly 600 Gauss. Above the working fluid, we float another freshwater layer with 0.8 cm depth, and the two layers exhibit a sharp interface. By passing a DC electric current laterally through the working fluid, we can produce a quasi-two-dimensional weakly turbulent flow by the resulting Lorentz body force. The current is large enough to produce weak turbulence and spatiotemporal dynamics but small enough to prevent any noticeable three-dimensional motion.<sup>38</sup>

The working fluid is seeded with 51  $\mu\text{m}$ -diameter fluorescent polystyrene particles. The Stokes number is of the order of  $10^{-4}$ , which means that the tracers are small enough to track the flow accurately.<sup>41</sup> The tracer's density is between those of freshwater and saltwater, so they remain at the interface. Since these two layers are miscible, there is no surface tension between tracers.

We use an IDT MotionPro M5 camera to image the particles illuminated with blue LED light sources. The sensor with a dimension of  $2320 \times 1728$  pixels measures a  $31.4 \times 23.4 \text{ cm}$  area (74 pixels/cm) at the center of the apparatus. About 30 000 particles can be captured at a rate of 60 frames per second. The combination of the particle seeding density and frame rate enables the highly spatiotemporally resolved Eulerian velocity fields because in two-dimensional flow, most of the flow dynamics happen above the energy injection length scale  $L_m$  due to the inverse cascade, which transfers energy to larger scales rather than smaller scales.<sup>42</sup> With our seeding density, the average distance

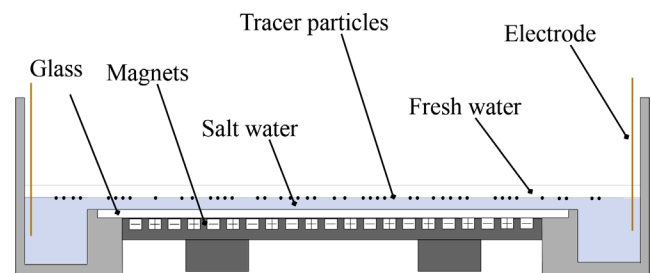


**FIG. 1.** Schematic of the experimental setup. The cross section of the experimental setup showing the two layers of fluid and tracer particles (not to scale) at the interface. The pair of electrodes conduct DC horizontally through the salt water. The vertical magnetic field from the permanent magnets interacts with the horizontal DC to generate the Lorentz force on the fluid that is nearly entirely in the plane. The interface between fresh water and salt water is the two-dimensional space we study. The thickness of the interface is roughly 1–2 mm while the lateral dimension of the interface is nearly a meter, so the interface has a very high aspect ratio that can be considered as quasi-two-dimensional.

between neighbor particles  $d$  is one order of magnitude smaller than the energy injection length scale  $L_m$ . In addition, the frame rate is two orders of magnitude smaller than the eddy turnover time (2.5 s, as is shown at the end of this section). We project the measured flow field onto a basis of numerically computed stream function eigenmodes to ensure a reliable two-dimensionality.<sup>38</sup> Furthermore, we interpolate the measured fluid properties (velocity, vorticity, and strain rate) onto regular Eulerian grids through cubic interpolation with grid size  $\Delta x = 10$  pixels (0.14 cm), which is of a similar order of magnitude as  $d$ .

Here, the two-dimensional weakly turbulent flow that we use in this work has a bulk Reynolds number  $Re = UL_m/\nu$  of 260, where  $U = 1.03$  cm/s is the root mean square velocity of the flow field. The eddy turnover time  $T_e = L_m/U$  is hence 2.5 s.

## B. Hybrid experimental-numerical model

We feed the flow field measured above with virtual swimmers. Swimmers are modeled as inertialess, noninteracting, point-like prolate ellipsoids. We assume one-way coupling;<sup>24,26</sup> consequently, the motion of the swimmers would not modify the flow field. In our simulation, the swimmers do not interact with each other. Swimmers are advected by a local flow velocity  $\mathbf{u}$  while also swimming with a constant intrinsic velocity  $v_s$  along the direction of the swimmer's major axis  $\mathbf{p}$ . The angular velocities of swimmers are determined by both local vorticity and strain rate following Jeffery's equation.<sup>14,25,26,43</sup> The local flow velocity, vorticity, and strain rate are obtained through cubic interpolation of the velocity, vorticity, and strain rate fields measured in Sec. II A. The full governing equations for the motion of swimmers are then

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{u}(\mathbf{x}, t) + v_s \mathbf{p}, \\ \dot{\mathbf{p}} &= \frac{1}{2} \boldsymbol{\omega} \times \mathbf{p} + \alpha \mathbf{p} \cdot \mathbf{E} \cdot (\mathbf{I} - \mathbf{p}\mathbf{p}),\end{aligned}\quad (1)$$

where the overdot represents the Lagrangian derivative,  $\boldsymbol{\omega} = \nabla \times \mathbf{u}$  is the flow vorticity,  $\mathbf{E} = (\nabla \mathbf{u} + \nabla \mathbf{u}^T)/2$  is the strain-rate tensor,  $\mathbf{I}$  is the identity matrix, and  $\alpha = (\gamma^2 - 1)/(\gamma^2 + 1)$  represents the eccentricity of the swimmer, where  $\gamma$  is the aspect ratio (ratio of the major and minor axis). When  $\alpha = 0$ , the particle is spherical, and when  $\alpha = 1$ , the particle is an infinitely thin line segment. Essentially, Jeffery's equation

states that swimmers with a higher eccentricity will have a stronger reorientation response to the local strain rate field, and the swimmers will also be reoriented according to the local vorticity field.

The motions of swimmers are integrated using a second-order Runge–Kutta scheme implemented in Matlab following the equations of motion (1). The time step of integration  $\Delta t$  is the same as the time between two frames of the experimental measurement, that is, 1/60 s. Since the Courant number  $C = U\Delta t/\Delta x = 0.13$  is much smaller than 1, the time step is small enough to avoid any overshoot problem. Our simulation only uses the center portion of the experimental measurement domain that spans  $23.2 \times 15.1$  cm for quantitative analysis. We initiate 20 000 swimmers in the simulation domain with random positions and directions. The swimmers are free to leave the domain. To keep the number of swimmers stationary, we replenish swimmers randomly in the frame surrounding the simulation domain every 10 time steps. On average, it takes a few eddy turnover times for a swimmer to leave the simulation domain; therefore, the simulation domain is large enough to statistically characterize the alignment and distribution properties of the virtual swimmers. The simulation will reach a statistically stationary state after about three eddy turnover times, where the statistics of the angle alignment and concentration distribution fluctuate near a constant value.

## C. Calculation of the finite-time Lyapunov exponent (FTLE)

Since the introduction of the term LCSs by Haller and Yuan,<sup>44</sup> many methods have been proposed to detect LCSs.<sup>45</sup> Here, we use FTLE to detect LCSs. This method has been previously applied to various flows to identify LCSs including two-dimensional flow,<sup>46–48</sup> turbulent boundary layer,<sup>49</sup> fully turbulent channel,<sup>50</sup> turbulent jet,<sup>51</sup> and ocean flows.<sup>52</sup> The FTLE was shown to be robust even with large velocity errors as long as those errors are small in a special time-weighted norm.<sup>53</sup>

The computation of the FTLE follows the method by Haller,<sup>45</sup> which is briefly summarized here. All of our following calculations are based on two-dimensional flows. The flow gradient  $\nabla F_{t_0}^t(\mathbf{x}_0)$  is calculated using a finite-difference approximation as

$$\nabla F_{t_0}^t(\mathbf{x}_0) \approx \begin{pmatrix} \frac{x(t; t_0, \mathbf{x}_0 + \boldsymbol{\delta}_1) - x(t; t_0, \mathbf{x}_0 - \boldsymbol{\delta}_1)}{2|\boldsymbol{\delta}_1|} & \frac{x(t; t_0, \mathbf{x}_0 + \boldsymbol{\delta}_2) - x(t; t_0, \mathbf{x}_0 - \boldsymbol{\delta}_2)}{2|\boldsymbol{\delta}_2|} \\ \frac{y(t; t_0, \mathbf{x}_0 + \boldsymbol{\delta}_1) - y(t; t_0, \mathbf{x}_0 - \boldsymbol{\delta}_1)}{2|\boldsymbol{\delta}_1|} & \frac{y(t; t_0, \mathbf{x}_0 + \boldsymbol{\delta}_2) - y(t; t_0, \mathbf{x}_0 - \boldsymbol{\delta}_2)}{2|\boldsymbol{\delta}_2|} \end{pmatrix},$$

where  $F_{t_0}^t(\mathbf{x}_0) = \mathbf{x}(t; t_0, \mathbf{x}_0)$  is the flow map that depicts the advection of a fluid element from an initial position  $\mathbf{x}_0$  at time  $t_0$  to its current position at time  $t$ ; and  $\mathbf{x}(t; t_0, \mathbf{x}_0) = [x(t; t_0, \mathbf{x}_0) \ y(t; t_0, \mathbf{x}_0)]$  is a vector. The  $\boldsymbol{\delta}_1$  and  $\boldsymbol{\delta}_2$  are two vectors pointing in horizontal ( $x$ ) and vertical ( $y$ ) directions, respectively, that represent a small perturbation, and both of their length are set to 10 pixels (0.14 cm) in our work. The right Cauchy–Green strain tensor defined as<sup>54</sup>

$$C(\mathbf{x}_0) = [\nabla F_{t_0}^t(\mathbf{x}_0)]^T \nabla F_{t_0}^t(\mathbf{x}_0),$$

which depicts how an initially infinitesimal perturbation at  $\mathbf{x}_0$  evolves with time. The eigenvalues  $\lambda_i(\mathbf{x}_0)$  and eigenvectors  $\xi_i(\mathbf{x}_0)$  of this symmetric and positive definite tensor satisfy

$$C\xi_i = \lambda_i\xi_i, \quad i = 1, 2; \quad 0 < \lambda_1 < 1 < \lambda_2, \quad \xi_1 \perp \xi_2.$$

The FTLE is then calculated as

$$FTLE(\mathbf{x}_0) = \frac{1}{t - t_0} \log \sqrt{\lambda_2(\mathbf{x}_0)}.$$

The forward time FTLE is calculated by integrating in forward time ( $t_0 < t$ ), and the backward time FTLE is calculated by integrating in backward time ( $t_0 > t$ ). The integration time length in this work is 5 s, which equals to  $2 T_e$ . We find that the FTLE is robust, and the result of this paper holds if integration time is between 1.5 and 3 eddy turnover time. Moreover, our previous work has shown that the FTLE is robust even with a substantial amount of downsampling of the measured velocity field;<sup>55</sup> hence our experimental resolution is fine enough in this work.

### III. PREFERENTIAL ALIGNMENT OF NON-SPHERICAL SWIMMERS NEAR ATTRACTING LAGRANGIAN COHERENT STRUCTURES

We start by investigating the instantaneous angle alignment of passive and rod-like ( $v_s = 0$  and  $\alpha = 1$ ) swimmers that are in the vicinity of the LCSs (repelling and attracting) after simulation reaches a statistically stationary state. For the simplicity of naming, we still call particles with no mobility as swimmers; and we call all particles with high aspect ratios as rod-like swimmers. The specific  $v_s$  and  $\alpha$  values would be clarified in brackets every time a kind of swimmer is mentioned. The Finite-Time Lyapunov Exponent (FTLE) approach is used to identify LCSs, and the calculation of FTLE fields is briefly introduced in Sec. II C and is detailed elsewhere.<sup>44,55–57</sup> In general, regions with extreme FTLE values are typically identified as FTLE ridges, and forward time FTLE (denoted as  $FTLE^+$ ) ridges and backward time FTLE (denoted as  $FTLE^-$ ) ridges are considered as effective detection of repelling and attracting LCSs.<sup>47,57</sup>

To characterize the angle alignment, the statistics of angle difference  $\theta$  between the major axis ( $\mathbf{p}$ ) of swimmers and the directions of local minimum FTLE gradient ( $\hat{\mathbf{e}}_{\min}$ ) are calculated. Figure 2 shows a  $FTLE^-$  field with the  $\hat{\mathbf{e}}_{\min}$  plotted in black arrows. In the vicinity of the LCS,  $\hat{\mathbf{e}}_{\min}$  is parallel to the nearest LCS, so  $\theta$  characterizes the angle

alignment between swimmers and the nearest LCSs. In two-dimensions,  $\theta$  ranges from 0 to  $\pi/2$ . Swimmers are initiated with random directions, which leads to an initial mean angle difference of  $\pi/4$ .

We identify the swimmers near the LCSs by adding thresholds to the FTLE field, and only swimmers with local FTLE absolute magnitude larger than a certain threshold are considered to be in the vicinity of the LCSs.<sup>11</sup> In this way, we can identify the ridges of FTLE field, which characterize the areas near the LCSs.<sup>47,57</sup> We test with several different thresholds of  $FTLE^+$  ( $FTLE^-$ ) to specify the alignment effects with different repelling (attracting) strengths near the LCSs, and we see that the result is not affected by the specific threshold we choose [Figs. 3(a) and 3(b)]. In the rest of this work, we use the absolute magnitude of 0.5 as the threshold, and our results are robust for any reasonable thresholds.

Passive rod-like swimmers ( $v_s = 0$ ,  $\alpha = 1$ ) show a much stronger alignment with  $FTLE^-$  [Fig. 3(b)] than with  $FTLE^+$  [Fig. 3(a)]. Near attracting LCSs (small  $FTLE^-$  values), swimmers with  $\theta$  smaller than the initial mean angle difference ( $\pi/4$ ) are two orders of magnitude more than swimmers with  $\theta$  greater than  $\pi/4$ . This means almost all swimmers are reoriented to align with the local  $\hat{\mathbf{e}}_{\min}$  and, thus, parallel to the nearby attracting LCSs.  $FTLE^+$ , however, shows a weaker effect on reorienting swimmers.

Khurana and Ouellette found that the elongated swimmers tend to be attracted to the repelling LCSs in an oscillating cellular flow.<sup>23</sup> However, we found that the swimmers' interaction with attracting LCSs dominates in two-dimensional weakly turbulent flows. In addition, this observed alignment with attracting LCSs is consistent with the findings by Ni *et al.*<sup>58</sup> The work by Ni *et al.* studies the alignment of passive rods and vorticity with Lagrangian stretching in a three-dimensional isotropic turbulent flow. They conclude that both the passive rods and vorticity independently align with the strongest Lagrangian stretching direction that is characterized by the eigenvectors corresponding to the largest eigenvalues of the left Cauchy–Green strain tensors. Our work uses the Right Cauchy–Green strain tensor to calculate the FTLE. The eigenvalues of the left and right Cauchy–Green strain tensors are the same, but the eigenvectors are, in general, not parallel because the eigenvectors of the left Cauchy–Green strain tensors show the direction of stretching in the coordinate system at the end of the integration, while the right Cauchy–Green tensors show the direction of stretching in the coordinate system at the beginning of the integration. Physically, both the largest eigenvector in Ni *et al.* and the  $\hat{\mathbf{e}}_{\min}$  of  $FTLE^-$  in our work represents the direction of strongest Lagrangian stretching in the forward time. Hence these two results qualitatively come to a consensus. Interestingly, vorticity in two-dimensional flow points out of the two-dimensional plane, so it will never be aligned with the strongest stretching direction. However, the non-spherical particles align with the strongest stretching direction in both two-dimensional and three-dimensional cases. This suggests that the vorticity only plays a minor role in re-orientating the non-spherical particles, which is consistent with both works by Ni *et al.* and us. More recently, Jayaram *et al.*<sup>59</sup> found similar alignment effects for spheroids in unsteady Taylor–Green vortex flow as Ni *et al.*,<sup>58</sup> which further strengthens this result.

One thing to note is that, in close vicinity of repelling LCSs (large  $FTLE^+$  values), there are two peaks in the probability density function (PDF), and the secondary peak can be due to the hyperbolic points where attracting LCSs cross the repelling LCSs.<sup>57</sup> Since passive

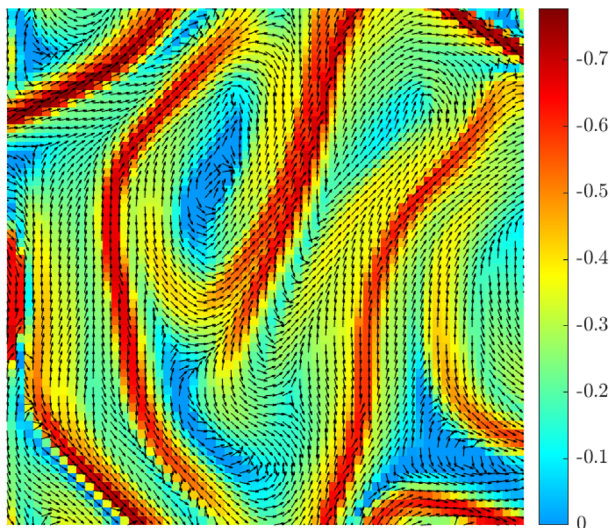


FIG. 2.  $FTLE^-$  field with arrows representing the directions of local minimum gradients  $\hat{\mathbf{e}}_{\min}$ . In the vicinity of the LCS,  $\hat{\mathbf{e}}_{\min}$  is parallel to the nearest LCS.

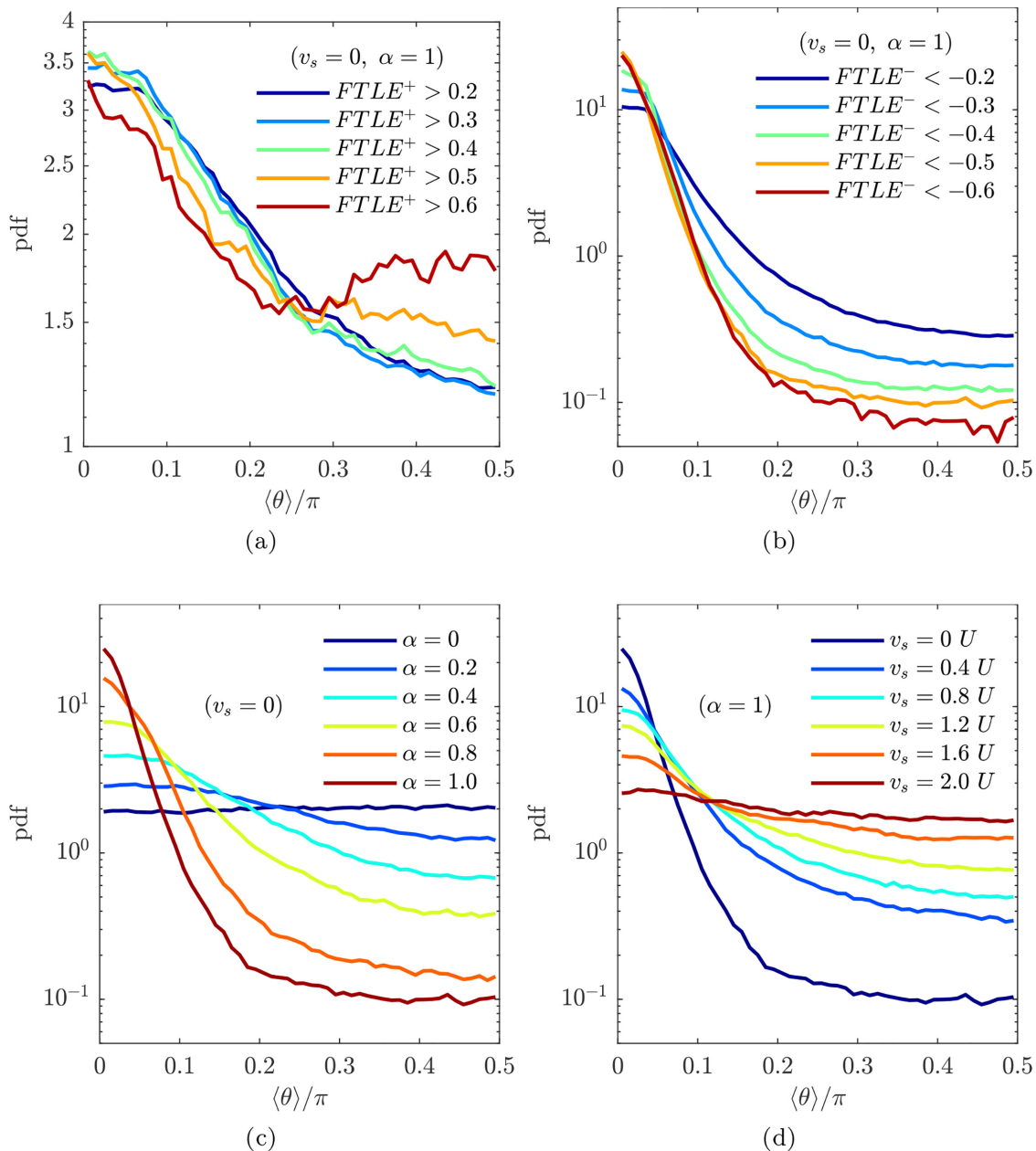


**FIG. 3.** (a) PDF of angle difference between  $\mathbf{p}$  of passive and rod-like ( $v_s = 0, \alpha = 1$ ) swimmers and the  $\hat{\mathbf{e}}_{\min}$  of  $FTLE^+$  at regions where  $FTLE^+$  is greater than the specified thresholds (shown in the legend). (b) PDF of angle difference between  $\mathbf{p}$  of passive and rod-like ( $v_s = 0, \alpha = 1$ ) swimmers and the  $\hat{\mathbf{e}}_{\min}$  of  $FTLE^-$  in regions where  $FTLE^-$  is smaller than the specified thresholds (shown in the legend). The unit of FTLE is  $s^{-1}$ . (c) PDF of the angle difference between  $\mathbf{p}$  of passive ( $v_s = 0$ ) swimmers with different eccentricity  $\alpha$  and the  $\hat{\mathbf{e}}_{\min}$  of  $FTLE^-$  near attracting LCSs. (d) PDF of the angle difference between  $\mathbf{p}$  of rod-like ( $\alpha = 1$ ) active swimmers with different  $v_s$  and the  $\hat{\mathbf{e}}_{\min}$  of  $FTLE^-$  near attracting LCSs.

rod-like swimmers have a much stronger alignment with attracting LCSs, the swimmers tend to be parallel with the attracting LCSs and, thus, perpendicular to the repelling LCSs at the hyperbolic points.

In the rest of the paper, we will focus on swimmers' interaction with attracting LCSs only due to its more substantial effects. With a smaller eccentricity, the passive rod-like swimmers ( $v_s = 0, \alpha > 0$ ) will

have a weaker response to the strain rate field, resulting in a weaker alignment with the attracting LCSs [Fig. 3(c)]. The passive spherical ( $v_s = 0, \alpha = 0$ ) swimmers' angular motion will only depend on the flow vorticity and totally lose the preferential alignment with the attracting LCSs. Hence, the PDF is uniform [Fig. 3(c)]. For active rod-like swimmers ( $v_s > 0, \alpha = 1$ ), increasing  $v_s$  would also cause a

weaker alignment because swimmers spend less time within local flow structures. For instance, when  $v_s = 2U$ , the flow's influence on the swimmers' motions will be very weak, and the PDF of the angle alignment becomes almost uniform [Fig. 3(d)], which is expected if swimmers are randomly oriented.

The preferential alignment effect near attracting LCSs naturally leads to the question of how does the alignment happens. Here, we propose at least a partial answer: that the alignment is due to the competition between the intrinsic mobility of the swimmers and the reorientation ability of the strain rate near the attracting LCSs. To test this idea, we consider not only the instantaneous alignment between  $\hat{\mathbf{e}}_{\min}$  and  $\mathbf{p}$  but rather the alignment between these two vectors computed at different times along the same Lagrangian trajectory of the swimmer. We define a lagged Lagrangian angle alignment as

$$\langle \theta(\tau) \rangle = \langle \cos^{-1}[\hat{\mathbf{e}}_{\min}(t) \cdot \mathbf{p}(t + \tau)] \rangle, \quad (2)$$

where  $t$  represents any moment after the simulation reaches a statistically stationary state, and the average is taken over an ensemble of many swimmer trajectories. Here,  $\tau$  represents the time lag between the two vectors, and  $\tau=0$  recovers the instantaneous alignment between  $\hat{\mathbf{e}}_{\min}$  and  $\mathbf{p}$  at the same position near an attracting LCS. We only study the swimmer trajectories that are in the vicinity of the attracting LCSs at  $\tau=0$ . Simulations are conducted with  $\alpha=1$  and with different  $v_s$  values.

Figure 4(a) shows the  $\langle \theta(\tau) \rangle$  for swimmers that are near the attracting LCSs at  $\tau=0$ . In all cases, we see that the  $\langle \theta(\tau) \rangle$  equilibrates to  $\pi/4$  as  $\tau$  increases from 0; that is, the swimmers gradually lose their memory as leaving the current attracting LCSs. Intriguingly, when we look at the negative  $\tau$  alignments of the swimmers within  $2T_e$ ,

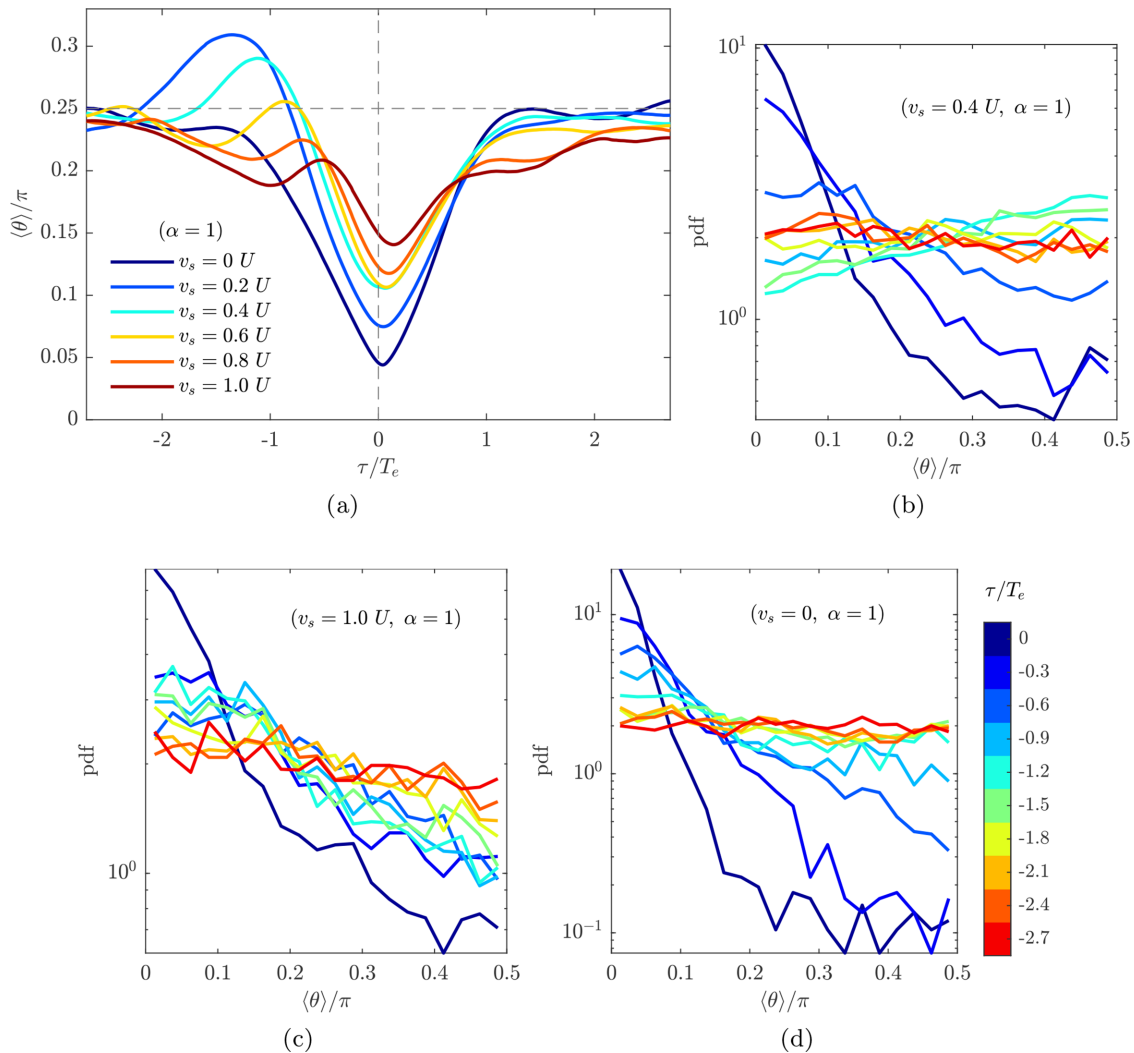


**FIG. 4.** (a) Lagged Lagrangian angle alignment  $\langle \theta(\tau) \rangle$  for rod-like swimmers ( $\alpha=1$ ) near attracting LCSs at  $\tau=0$ . (b) PDF of  $\langle \theta(\tau) \rangle$  for swimmers with  $v_s = 0.4U$  that are near attracting LCSs at  $\tau=0$ . (c) PDF of  $\langle \theta(\tau) \rangle$  for swimmers with  $v_s = 1.0U$  that are near attracting LCSs at  $\tau=0$ . (d) PDF of  $\langle \theta(\tau) \rangle$  for swimmers with  $v_s = 0$  that are near attracting LCSs at  $\tau=0$ . For (b)–(d), from red to blue, the  $\tau$  ranges from  $-2.7T_e$  to  $0.7T_e$  with equal time spacing.

swimmers with an intermediate velocity ( $v_s \in [0.2U, 0.6U]$ ) show a mean angle difference  $\langle \theta \rangle$  greater than  $\pi/4$ , which means that the attracting LCSs preferentially capture swimmers that were more perpendicular to them before they reach the attracting LCSs. For fast swimmers ( $v_s > 0.6U$ ), mean angle difference  $\langle \theta \rangle$  is smaller than  $\pi/4$  for all negative  $\tau$ , which indicates that only fast swimmers that are initially more parallel to the nearest LCSs can be captured by the LCSs.

To look at the details of the angle alignments along the swimmers' trajectories before they arrive in the vicinity of the attracting LCSs, we calculate the full PDFs of  $\langle \theta(\tau) \rangle$  at different  $\tau$  in addition to the ensemble mean value. Figures 4(b)–4(d) correspond to the simulations for  $v_s = 0.4U$ ,  $v_s = 1.0U$ , and  $v_s = 0$ , respectively, and all simulations have  $\alpha = 1$ . Figures 4(b)–4(d) show the time development of the PDF of  $\langle \theta(\tau) \rangle$  with  $\tau$  values from  $-2.7T_e$  to  $0T_e$ , and each color corresponds to a single  $\tau$ . For swimmers with intermediate  $v_s$  [ $v_s \in [0.2U, 0.6U]$ , Fig. 4(b)], the PDFs are originally evenly distributed (red lines). As time progresses, it peaks at  $\pi/2$  (light green lines) before peaking at 0 (blue lines). Physically speaking, the swimmer that will eventually arrive in the vicinity of the attracting LCSs at  $\tau = 0$  has no correlation with the corresponding attracting LCSs at  $\tau = -2T_e$  so the lagged angle alignment PDF is evenly distributed. As the time progresses, the PDF of the lagged angle alignment peaks at  $\pi/2$  indicating that for the swimmers that eventually reach the attracting LCSs at  $\tau = 0$ , the major portion of them were more perpendicular to the attracting LCSs when they started to be correlated with the attracting LCSs. This is because the swimmers are continuously reoriented to be parallel to the attracting LCSs as they are approaching attracting LCSs, and  $v_s$  components perpendicular to the attracting LCSs are reduced due to the reorientation effect. Once a swimmer is parallel to the local attracting LCSs, it will never reach it since both local velocity and intrinsic velocity are parallel to the nearby LCSs. Figure 5(a) shows the schematic that compares the trajectories of two swimmers with the same intermediate intrinsic velocity and eccentricity while different initial angles relative to the attracting LCS. Therefore, for swimmers with an intermediate velocity ( $v_s \in [0.2U, 0.6U]$ ), the reorientation ability of the strain rate near the attracting LCSs dominates the intrinsic mobility of the swimmers, and only the swimmers that are more perpendicular to the attracting LCSs can overcome the continuous reorientation effect while approaching the attracting LCSs.

If the swimmers swim fast, attracting LCSs would, however, preferentially capture swimmers orienting more tangential to the attracting LCSs [indicated by PDFs peaking at the angle of 0 at all  $\tau$ , Fig. 4(c)]. Physically, faster swimmers move through the local flow structures with smaller time scales because of their high intrinsic mobility, and the intrinsic mobility of the swimmers dominates the reorientation ability of the strain rate near the attracting LCSs. So, only swimmers that are initially more tangential with the nearby attracting LCSs will eventually be reoriented and captured at the attracting LCSs within short time scales, and swimmers originally more perpendicular to the attracting LCSs will directly swim through the attracting LCSs. Figure 5(b) shows the schematic that compares the trajectories of two fast swimmers with different initial angles relative to the attracting LCS. Passive swimmers ( $v_s = 0$ ) are the same as the flow tracers and are the control case here [Fig. 4(d)].

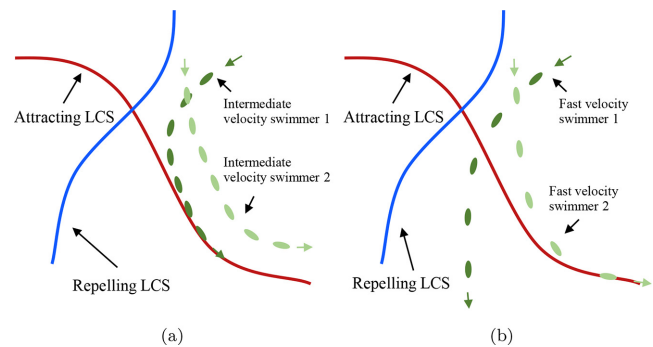


**FIG. 5.** (a) Schematic of trajectories for swimmers with intermediate intrinsic velocity. The consecutive swimmer locations along a trajectory are with an equal time gap, and the arrows indicate the direction of swimmers' velocities. In this case, the reorientation ability of the strain rate near the attracting LCS dominates the swimmer's mobility, and a swimmer must be more perpendicular to the attracting LCS to eventually reach and align with it. Otherwise, a swimmer will be reoriented to be parallel to the attracting LCS before reaching it due to the strong reorientation ability of the strain rate. Swimmer 1 was initially more perpendicular to the attracting LCS, and swimmer 2 was more tangential to the attracting LCS. Swimmer 1 overcomes the reorientation effect and eventually arrives at the attracting LCS, while swimmer 2 was reoriented to be parallel to the attracting LCS before reaching it and hence can never reach the attracting LCS. (b) Schematic of trajectories for swimmers with fast intrinsic velocity. The consecutive swimmer locations along a trajectory are with equal time gap and the arrows indicate the direction of swimmers' velocities. In this case, the swimmer's mobility dominates the reorientation ability of the strain rate near the attracting LCS, and a swimmer must be more parallel to the attracting LCS to eventually align with it. Otherwise, the swimmer will penetrate the attracting LCS due to the strong mobility. Swimmer 1 is initially more perpendicular to the attracting LCS, and swimmer 2 is more tangential to the attracting LCS. Swimmer 1 will directly pass the attracting LCS due to the limited reorientation effect of the attracting LCS, while swimmer 2 can be slightly reoriented to be captured by the attracting LCS.

#### IV. HETEROGENEOUS DISTRIBUTION OF NON-SPHERICAL SWIMMERS NEAR ATTRACTING LAGRANGIAN COHERENT STRUCTURES

Once the rod-like swimmers with an intermediate intrinsic velocity ( $v_s \in [0.2U, 0.6U]$ ) reach and are reoriented to be parallel to the attracting LCSs, they will swim along the attracting LCSs collectively. We hypothesize that the collective swim along the attracting LCSs will lead to accumulation and thus heterogeneous distribution, near the attracting LCSs. The strength of the accumulation will vary as a function of  $v_s$  and  $\alpha$ . To test this hypothesis, we conduct simulation for long enough to reach statistically stationary conditions for wide combinations of  $\alpha$  and  $v_s$ . Figures 6(a)–6(c) show snapshots of swimmer distribution for three typical cases. We see that only swimmers with intermediate intrinsic velocity ( $v_s \in [0.2U, 0.6U]$ ) and  $\alpha > 0.6$  exhibit strong heterogeneous distributions, which is characterized by the accumulation of swimmers near the attracting LCSs. [Fig. 6(a)]. The heterogeneous distribution disappears if we reduce the  $\alpha$  [Fig. 6(b)] or increase the  $v_s$  [Fig. 6(c)].

To quantify the accumulation of swimmers near the attracting LCSs with different  $\alpha$  and  $v_s$ , we define normalized concentration  $C$  to be the swimmer concentration near the LCSs normalized by the concentration that assumes all the swimmers in the simulation domain are uniformly distributed. Therefore,  $C$  higher than one indicates

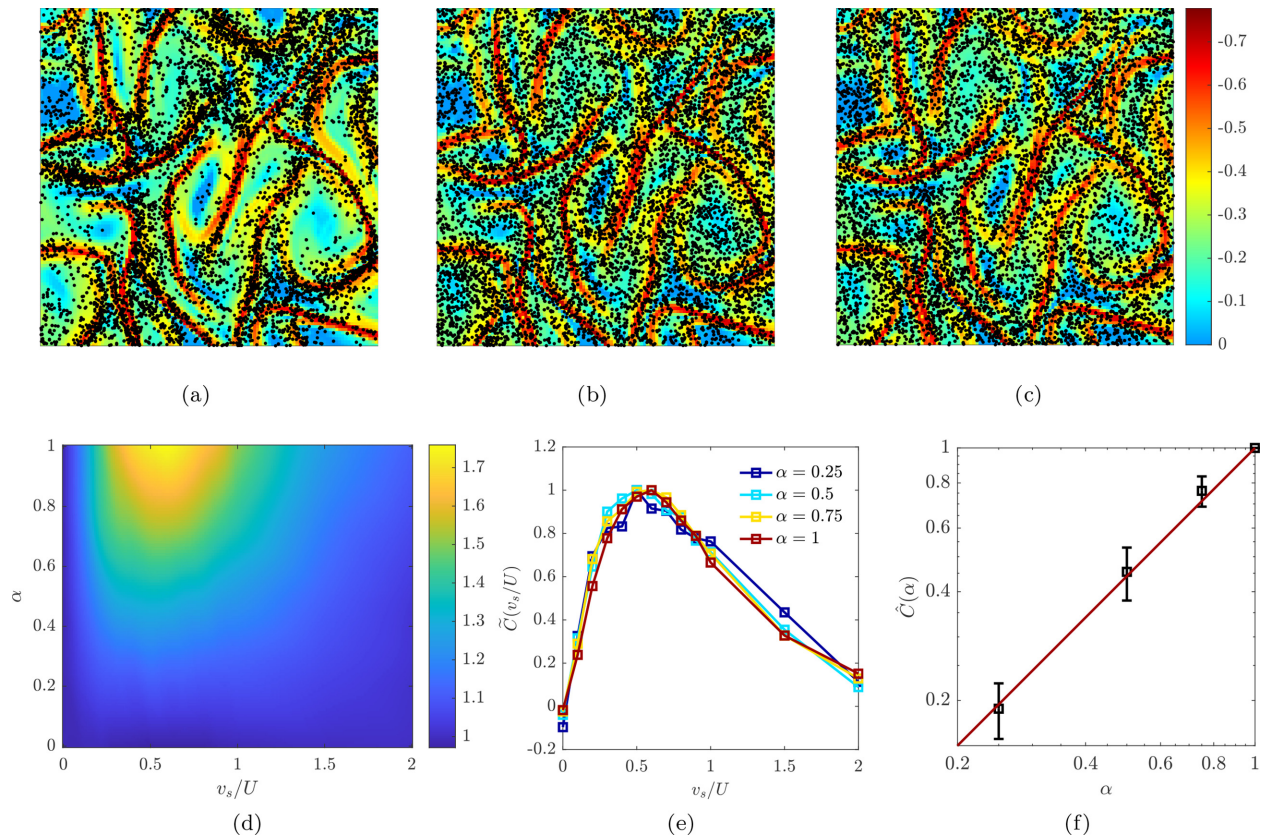


**FIG. 6.** (a) Swimmers with intermediate intrinsic velocity and rod-like shape ( $v_s = 0.5 U$ ,  $\alpha = 1$ ) distribution with  $\text{FTLE}^-$  field. (b) Swimmers with intermediate intrinsic velocity and rod-like shape ( $v_s = 0.5 U$ ,  $\alpha = 0.1$ ) distribution with  $\text{FTLE}^-$  field. (c) Swimmers with high intrinsic velocity and rod-like shape ( $v_s = 3 U$ ,  $\alpha = 1$ ) distribution with  $\text{FTLE}^-$  field. (d) The normalized concentration  $C$  of swimmers with  $\alpha$  ranging from 0 to 1 and  $v_s$  ranging from 0 to  $2U$  near attracting LCSs. (e) Self-similar  $C - v_s$  profile of swimmers with different  $\alpha$ . All profiles collapse into a single curve  $\tilde{C}$ , which is a function of  $v_s/U$ . (f) Power-law dependence of  $C(\alpha)$  on  $\alpha$ ; the black squares with error bars represent the mean and standard deviation of twelve normalized profiles with different  $v_s$  values ranging from  $0.1U$  to  $2U$ ; the red line represents the best fit with the exponent of 1.18.

accumulation and, conversely,  $C$  less than one indicates vacancy. Figure 6(d) maps the  $C$  near the attracting LCSs with  $v_s$  from 0 to  $2U$  and  $\alpha$  from 0 to 1. It clearly shows that the accumulation is intensified with an increase in  $\alpha$  with any given  $v_s$ . For swimmers with any given  $\alpha$ , there exists an intermediate  $v_s$  yielding the maximum accumulation near attracting LCSs. For slow swimmers, the accumulation effect is limited by the swimmer's intrinsic mobility  $v_s$ . For fast swimmers, however, the accumulation effect is limited by the limited reorientation ability of the strain rate near the attracting LCSs. Spherical swimmers show no noticeable accumulation effect since they do not respond to the strain rate. These observations are consistent with the result found in Sec. III.

Moreover, we find the self-similarity of the accumulation. We normalize the  $C(v_s/U, \alpha)$  by the maximum concentration of corresponding  $\alpha$ , and all the  $C - v_s$  curves collapse into a single curve. Thus, the normalized  $C$  is not a function of  $\alpha$  any more. Mathematically, we define

$$\tilde{C}\left(\frac{v_s}{U}\right) = \tilde{C}\left(\frac{v_s}{U}, \alpha\right) \equiv \frac{C\left(\frac{v_s}{U}, \alpha\right) - 1}{C_{\max}(\alpha) - 1}, \quad (3)$$

where  $C_{\max}(\alpha)$  is the maximum concentration for swimmers with the corresponding  $\alpha$ . The first equality in Eq. (3) is due to the observed collapse of the data indicated in Fig. 6(e). Moreover, the independence of  $\tilde{C}(v_s/U)$  on  $\alpha$  allows the  $C - \alpha$  curves with different  $v_s$  to also collapse onto a single curve that shows a power-law dependence on  $\alpha$ <sup>60</sup>

$$\tilde{C}(\alpha) \equiv \frac{C_{\max}(\alpha) - 1}{C_{\max}(\alpha = 1) - 1} = \frac{C(\alpha) - 1}{C(\alpha = 1) - 1} \Big|_{\frac{v_s}{U}}. \quad (4)$$

The second equality is based on the simulation data, which imply that the  $C(\alpha) - 1$  is power-law dependent on  $\alpha$  for any given  $v_s$  [Fig. 6(f)]. This reveals the self-similarity for the heterogeneous distribution of swimmers over a wide range in the parameter space, and the intricate swimmer distribution problem further reduces to only one independent parameter.

## V. CONCLUSIONS

Using a simple hybrid numerical-experimental model, in which we fed virtual swimmers to experimentally measured flow, we had revealed that the angle orientation and heterogeneous distribution of active non-spherical swimmers are closely related to a long-standing

flow structure, attracting LCSs. Our work answered, at least partially, three questions we raised in the introduction. First, we found that the alignment effect between swimmers and attracting LCSs is one order of magnitude stronger than that of repelling LCSs. Second, in addition to the steady state angle alignment statistics, we had investigated lagged angle alignment along the swimmer trajectories and had concluded that the alignment between swimmers and the attracting LCSs is the result of the competition between the intrinsic mobility of the swimmers and the reorientation ability of the strain rate. The intricate interaction can be rationalized to two key intrinsic parameters: body eccentricity and swimmer's motility. The alignment effect strengthens with larger body eccentricity and smaller swimming velocity. Meanwhile, the attracting LCSs preferentially capture swimmers with intermediate velocity that were initially more perpendicular to it and fast swimmers that were initially more tangential to it. We further found that as the consequence of the preferential alignment near attracting LCSs, swimmers with intermediate intrinsic velocities and large eccentricity can accumulate near the attracting LCSs and hence result in a heterogeneous distribution. Finally, we found the self-similarity of the accumulation of swimmers near the attracting LCSs, which simplifies this complicated problem to have a single independent parameter.

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## DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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